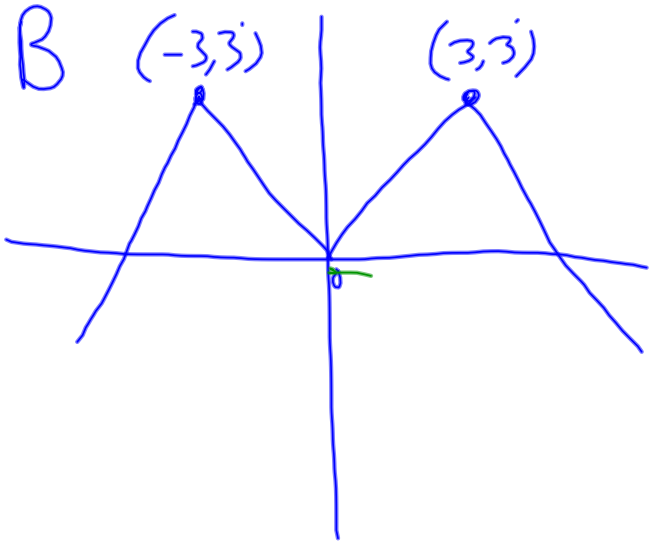
23)

B



$\lim_{x\to 0} \frac{f(x)-f(0)}{x} = 0$

Not true

$\lim_{x\to 0^+} f(x) = +$

$\lim_{x\to 0^-} f(x) = -$

2 sided limit DNE
b/c $x\to 0^+$ & $x\to 0^-$
do not agree

The simplest of complicated anti-derivatives

$$\int \frac{1}{x(x-1)(x-2)}\,dx =$$

Feb 8-10:55 AM

$$\frac{1}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-2)}$$

$$\frac{A(x-1)(x-2)+B(x)(x-2)+C(x)(x-1)}{x(x-1)(x-2)} = \frac{1}{x(x-1)(x-2)}$$

$$A(x^2-3x+2)+B(x^2-2x)+C(x^2-x) = 1$$

$$Ax^2-3Ax+2A+Bx^2-2Bx+Cx^2-Cx = 1$$

$$(A+B+C)x^2+(-3A-2B-C)x+2A = 0x^2+0x+1$$

$$A+B+C=0$$

$$-3A-2B-C=0$$

$$2A = 1 \quad \therefore A=\frac{1}{2}$$

$$B+C=-\frac{1}{2}$$

$$+\left(-2B-C=\frac{3}{2}\right)$$

$$-B = 1 \quad \therefore B=-1$$

$$(-1)+(C) = -\frac{1}{2} \quad \therefore C=\frac{1}{2}$$

$$\frac{1}{x(x-1)(x-2)} = \frac{1}{2}\left(\frac{1}{x}\right)-\left(\frac{1}{x-1}\right)+\frac{1}{2}\left(\frac{1}{x-2}\right)$$

Integration by the method of partial fractions

Feb 9-7:34 AM

$A(x-1)(x-2)+B(x)(x-2)+C(x)(x-1)=1$

When $x=0$ . . . .

$A(-1)(-2)+0+0=1$

$2A=1$ or $A=\frac{1}{2}$ $\quad A=\frac{1}{2}$

when $x=1$

$0+B(1)(-1)+0=1$ $\quad -B=1$ or $B=-1$

when $x=2$

$0+0+C(2)(1)=1$ $\quad 2C=1$ $\quad C=\frac{1}{2}$

Feb 9-7:34 AM

$$\frac{1}{2}(x-1)(x-2) - (x)(x-2) + \frac{1}{2}(x)(x-1) = 1$$

$$y_1 = \left(\frac{1}{2}\right)(x-1)(x-2) - (x)(x-2) + \left(\frac{1}{2}\right)(x)(x-1)$$

$$\int \frac{1}{x(x-1)(x-2)}\,dx = \frac{1}{2}\int \frac{1}{x}\,dx - \int \frac{1}{x-1}\,dx + \frac{1}{2}\int \frac{1}{x-2}\,dx$$

$u = x-1$
$du = dx$

$$= \frac{1}{2}\left(\ln|x|\right) - \ln|x-1| + \frac{1}{2}\ln|x-2| + C$$

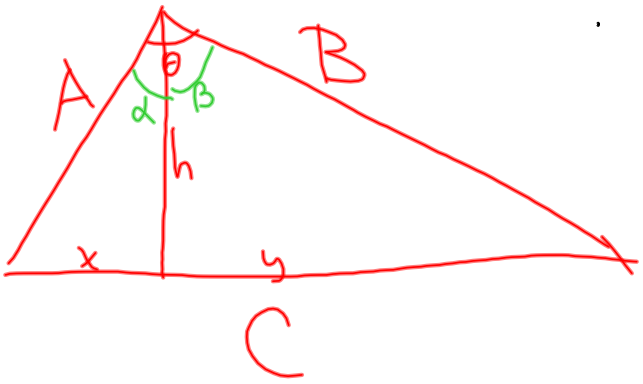
$$\int \frac{1}{x(x-4)(x+6)}\,dx$$

$$\frac{1}{x(x-4)(x+6)} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{x+6}$$

A: $-\frac{1}{24}$ $\quad B$: $+\frac{1}{40}$ $\quad C$: $+\frac{1}{60}$

Feb 9-8:22 AM

$C^2 = A^2 + B^2 - 2AB(\cos(\theta))$?



$\cos\alpha = \frac{h}{A}$

$\cos\beta = \frac{h}{B}$

$\sin\alpha = \frac{x}{A}$

$\sin\beta = \frac{y}{B}$

$\cos C = \cos(\alpha+\beta)$

$= \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{h^2}{AB} - \frac{xy}{AB}$

$A^2 - x^2 = h^2 = B^2 - y^2$

$A^2 = B^2 + x^2 - y^2$

$C^2 = (x+y)^2 = x^2 + y^2 + 2xy$

$C^2 = B^2 - A^2 + x^2 + 2xy$

$\cos C = \frac{h^2 - xy}{AB}$

$h^2 - xy = AB\cos C$

$2xy = 2h^2 - 2AB\cos C$

$C^2 = B^2 - A^2 + x^2 + 2h^2 - 2AB\cos C$

$C^2 = B^2 + h^2 - 2AB\cos C$

$C^2 = B^2 + A^2 - x^2 - 2AB\cos C$

Feb 8-7:36 AM

The simplest of complicated anti-derivatives

$$\int \frac{1}{x(x-1)(x-2)}\,dx =$$

Feb 8-10:55 AM

First .... some precalculus

$$\frac{1}{x(x-1)(x-2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-2}$$

$$\frac{A(x-1)(x-2) + B(x)(x-2) + C(x)(x-1)}{(x)(x-1)(x-2)} = \frac{1}{x(x-1)(x-2)}$$

$$A(x-1)(x-2) + B(x)(x-2) + C(x)(x-1) = 1$$

$$A(x^2-3x+2) + B(x^2-2x) + C(x^2-x) = 1$$

$$(A+B+C)x^2 + (-3A-2B-C)x + 2A = 0x^2 + 0x + 1$$

$$A+B+C = 0$$

$$-3A-2B-C = 0$$

$$2A = 1 \quad \therefore A = \frac{1}{2}$$

$$B+C = -\frac{1}{2}$$

$$+\left(-2B-C = \frac{3}{2}\right)$$

$$-B = 1 \quad \therefore B = -1$$

$$-1 + C = -\frac{1}{2} \quad \therefore C = \frac{1}{2}$$

$$\frac{1}{x(x-1)(x-2)} = \frac{1}{2}\frac{1}{x} - \frac{1}{x-1} + \frac{1}{2}\frac{1}{x-2}$$

Feb 8-7:36 AM

$A(x-1)(x-2)+B(x)(x-2)+C(x)(x-1)=1$

if $x=1$

$0+B(1)(-1)+0=1$

$-B=1$ and $B=-1$

if $x=0$

$A(-1)(-2)+0+0=1$

$2A=1$ - - - - - - - $A=\frac{1}{2}$

if $x=2\cdots$ $C(2)(1)=1$ or $C=\frac{1}{2}$

Feb 9-11:32 AM

back to CALCULUS

$$\int \frac{1}{x(x-1)(x-2)}\,dx = \frac{1}{2}\int \frac{1}{x}\,dx - \int \frac{1}{x-1}\,dx + \frac{1}{2}\int \frac{1}{x-2}\,dx$$

$u = x-1$, $du = dx$

$$= \frac{1}{2}\ln|x| - \ln|x-1| + \frac{1}{2}\ln|x-2| + C$$

Feb 9-11:32 AM

$$\frac{1}{(x-2)(x-3)(x-\pi)} = \frac{A}{x-2} + \frac{B}{x-3} + \frac{C}{x-4}$$

A	B	C
$\frac{1}{2}$	-1	$\frac{1}{2}$

$$A(x-3)(x-4) + B(x-2)(x-4) + C(x-2)(x-3) = 1$$

$$\frac{A}{x-2} + \frac{B}{x-3} + \frac{C}{x+4}$$

$$A(x-3)(x+4) + B(x-2)(x+4) + C(x-2)(x-3) = 1$$

A	B	C
$-\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{42}$

Feb 9-11:32 AM